Synchronization of complex networks with nonidentical nodes

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Abstract. In this communication we present the synchronization of complex networks adding a derivative coupling term in the network equation. This is, using a simple derivative action the synchronous behavior of a complex network is achieved. We consider strictly different chaotic systems in nodes. We show that the derivative term leads to the synchronous behavior in networks that has three different dynamical models in nodes, whereas when there is no derivative term the network is leaded to an equilibrium point. Numerical simulation are provided to illustrate the result.
Keywords: Complex Networks, Synchronization, Chaos

1 Introduction

Network are everywhere in nature, a network can be seen as a set of objects connected or linked with some strength coupling. The study of this class of dynamical system has attracted a lot of attention see for instance[1],[2],[3]. Complex networks involves a common phenomenon between dynamical systems, synchronization [4], moreover synchronization of chaotic systems is still an open topic (refer to [5],[6],[7]). Therefore synchronization of complex networks is a challenging recent problem under study. Examples of networks are so diverse, individuals in a community, where every person is represented as a node; the internet, which is a set of routers connected by physical or virtual connections; the Web, where virtual web documents can be accessed via other web links [8], or others web documents can be accessed via this web. Biological networks, where an important issue is to understand the interaction between cells [9]; in protein interactions, it has been shown that this interaction is highly heterogeneous [10]; epidemic spreading studies [11]; until collaboration networks [12]; thus understanding the synchronization of complex networks is an essential issue in science and technology.

The problem of network synchronization has been studied departing from the determination of the appropriate coupling strength (see for instance [2],[13]), and assuming that every system in each node is equal to any other system in the network. However, these assumptions are not realistic, since the nodes in a network

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community, internet, webs etc. are in general different. In this sense we deal with the particular problem of synchronize a network which nodes are represented by nonidentical chaotic systems. Moreover, we seek for synchronization of the network in a chaotic attractor, neither into a limit cycle nor an equilibrium point. Maoyin and Donghua [14] reported the synchronization of a complex network assuming unknown the dynamics in each node and the strength coupling functions, authors used the LaSalle invariance principle and a simple linear controller. However, they assume that there is an isolated dynamics to which the nodes in the network are synchronized. This is a strong assumption, since the behavior of the network depends on the collective dynamics and not on an isolated node. Other approach to control and synchronization of complex network is provided in [15] where they considered a synchronization scheme assuming that a few nodes are controlled via a proportional term. Nevertheless, the synchronization objective was to stabilize the global behavior in an equilibrium point and not in a chaotic attractor. Gua-Ping and coworkers [16] reported an approach to synchronize a dynamical complex network using state observers, but the synchronization is achieved via solving a LMI, solution of this kind of inequalities requires a great computational capacities for networks with many nodes, which represents a consumption of resources.

We present an approach that consider a derivative coupling term in the network equation to improve the synchronous behavior. To this end, we propose to synchronize scale-free networks and small-world networks with nonidentical nodes. The derivative term lead the global behavior of the network to a chaotic attractor. Compared with the standard coupling, the derivative coupling under certain network topology reaches the chaotic synchronous behavior, whereas the standard coupling lead the network to the equilibrium ir a limit cycle.

The paper is organized as follows. In Section II the model for the complex networks dynamics is described, in Section III we propose the derivative term to improve the synchronization behavior, results on synchronization of scale-free and small-world networks are illustrated in Section IV and finally, the work is closed with some concluding remarks in section V.

2 Model of dynamical complex networks

Consider a dynamical complex network with N identical nodes and diffusive couplings, which every node is an identical n-dimensional dynamical system and with state equation given by

$$\begin{aligned} \dot{x}_i &= f(x_i) + c\textstyle\sum_{j=1}^{N} a_{i,j}\Gamma(x_j - x_i) \\ y_i &= x_i \end{aligned} \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \cdots, x_{in})^T \in \mathbb{R}^n$ is the state vector for the $i-th$ node, $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a smooth nonlinear vector field, $c > 0$ stands for the coupling strength, the constant matrix $\Gamma = diag(\gamma_1, \gamma_2, \ldots, \gamma_n)$ is a diagonal matrix with $\gamma_k = 1$ for the $k - th$ state, this means that two nodes are coupled via the $k - th$ state variable. In other words, matrix Γ determines by which variables the oscillators are coupled.

Now, the coupling coefficients $a_{i,j}$ are the incomes of a real matrix A, if it is a connection between node i and node j $(j \neq i)$, therefore $a_{ij} = a_{ji} = 1$; otherwise, $a_{i,j} = a_{j,i} = 0 (j \neq i)$. Then the coupling matrix is diagonal and irreducible if we consider that there are no isolated nodes, thus, we know that zero is an eigenvalue of A with multiplicity 1, and the others eigenvalues of A are strictly positive. Network synchronization is defined as follows

Definition 1. A complex network is Completely Synchronized if every node synchronizes each other, $lim_{t \to \infty} \parallel x_i - x_j \parallel \to 0$ for all $1 \leq i, j \leq N$.

2.1 The proposed derivative coupling

We consider that the vector fields $f(x_i)$ in every node of the network are in general nonidentical. This is a realistic consideration since in real networks dynamical system in a node is in general different. Therefore, the main contribution is the modification of the equation (1) by adding a derivative coupling term, with this new term the synchronization behavior of the network is investigated. With these modifications we can rewrite (1) as follows

$$\dot{x}_i = f(x_i) + c_P \sum_{j=1}^{N} a_{i,j} \Gamma (x_j - x_i) + c_D \sum_{j=1}^{N} a_{i,j} \Gamma (\dot{x}_j - \dot{x}_i) \qquad (2)$$

where we have added the derivative part, c_P and c_D are the Proportional and Derivative coupling strength respectively. The derivative term is such that the network dynamics is increased in the sense that the interconnection between nodes are provided by the time variation of the linking state. Therefore, the linking of the nodes in the network are composed by the states and the time derivative of the states. With this modification we look for the synchronization of complex networks in a chaotic attractor which is defined by the collective behavior of the network.

3 Results on synchronization

We seek for complete synchronization of a network in a synchronization manifold $\Psi(x)$, in other words, synchronization of the network in a chaotic attractor. Where the synchronization manifold is given by $\Psi(x) = x_1 = x_2 = \cdots = x_N$ and correspond to the synchronized behavior. It is clear that x_i for some i could be seen as a solution of an isolated system which in this case is uncertain. The synchronization manifold $\Psi(x)$ is a result of the collective behavior and it is not known a prior.

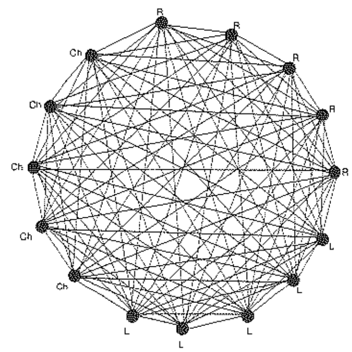

Fig. 1. Small-World complex network, where R, L, Ch stand for the Rössler, Lorenz and Chen systems respectively.

3.1 Synchronization of Small-World Networks

Small world networks are characterized by possessing a relatively small average path length. The average path length, is defined as the mean distance between two nodes, averaged over all pair of nodes. To illustrate the result, let us consider a network with three autonomous chaotic systems given by the Rössler, Chen and Lorenz systems

$$\begin{aligned} \dot{x}_1 &= -x_2 - x_3 \\ \dot{x}_2 &= x_1 + ax_2 \\ \dot{x}_3 &= (x_1 - d)x_3 + b \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{x_1} &= \sigma(x_2 - x_1) \\ \dot{x_2} &= \rho x_1 + \beta x_2 - x_1x_3 \\ \dot{x_3} &= x_1x_2 - \alpha x_3 \end{aligned} \tag{4}$$

$$\begin{aligned} \dot{x_1} &= s(x_2 - x_1) \\ \dot{x_2} &= rx_1 - x_1x_3 + x_2 \\ \dot{x_3} &= x_1x_2 - gx_3 \end{aligned} \tag{5}$$

Where the parameters for the system in node $i-th$, are different, which represents nonidentical dynamical systems. Thus, the network considered for this case is illustrated in Figure 1, where 5 Rössler systems, 5 Chen systems and 5 Lorenz systems were connected and with $\Gamma = diag(1,1,1)$.

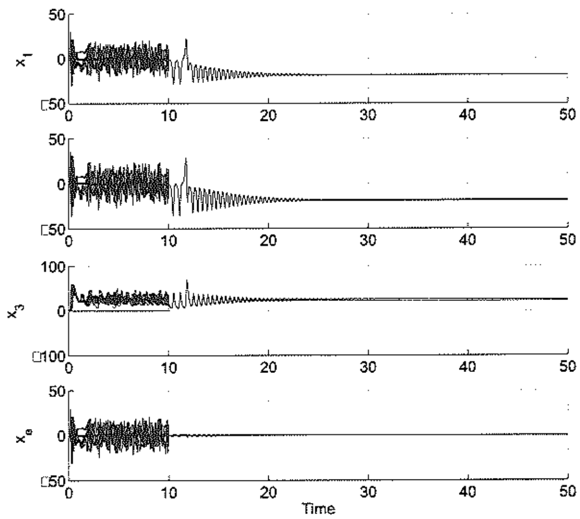

Fig. 2. Stabilization of the Small-World complex network at an equilibrium.

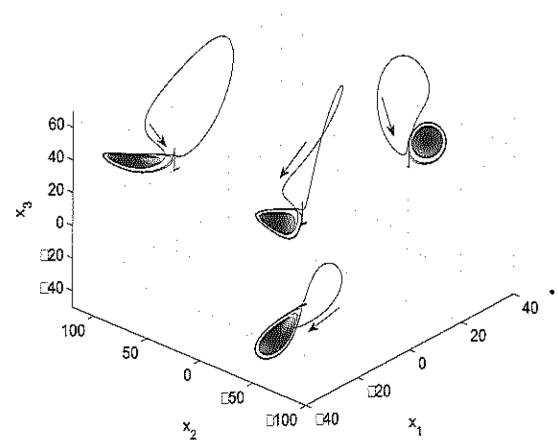

Fig. 3. Attractor for the stabilization of the network.

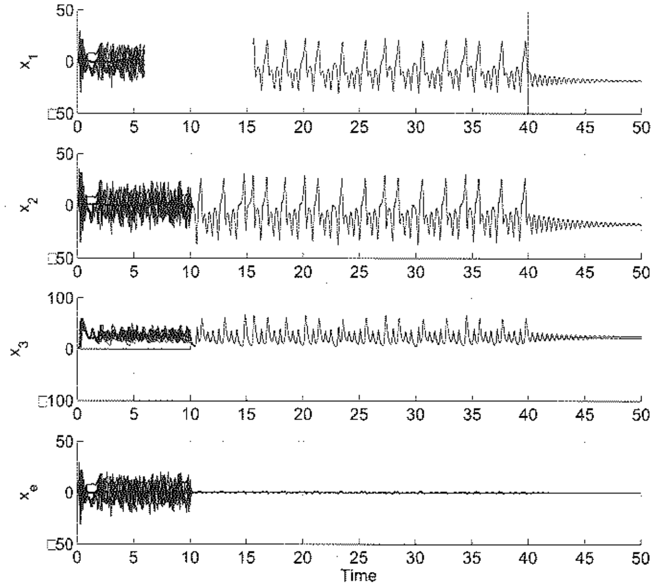

Fig. 4. Synchronization of the network.

The behavior of the network with no derivative coupling is illustrated in Figure 2, where the connection was activated at $t = 10$sec. It can be observe that the dynamics of the network is leaded to an equilibrium point. This means that with this network topology the synchronization is achieved but in an equilibrium, this is, the the collective behavior is such that inhibits the chaotic behavior in each node. It is important to note that there is no value for the coupling parameter c_P such that the network synchronizes in a chaotic attractor, for this case we use $c_P = 15$.

In Figure 3 the attractor of one node in the network and its corresponding canonical projections are illustrated. The trajectories of each system in the network are driven to an equilibrium point. A conjecture for this behavior can be the fact that the system in the network are strictly different, this means that, since each system in the network possess a strictly different vector fields and the corresponding trajectories are also different.

Therefore, in order to obtain chaotic synchronization in the network, we use the modified equation (2). Thus, considering the same network topology but with $c_P = 15$ and $c_D = 1$ the synchronization in the chaotic attractor is obtained. The time evolution is illustrated in Figure 4. Where we have connected the network at $t = 10$sec. using the derivative coupling. At $t = 40$sec. the derivative coupling is disconnected and the behavior is leaded again to the equilibrium point.

In Figure 5 the chaotic attractor of a single node is presented as well as its corresponding canonical projections. This attractor was obtained using the derivative coupling, and again after a period of time the derivative coupling is disconnected and the trajectory is leaded to an equilibrium point. Note that, in Figure 4, the corresponding error $x_e = x_i - x_j$ for $j = 1, 2, \ldots, N$, this is, the error of the output of node i-th, and the node j-th, for all j.

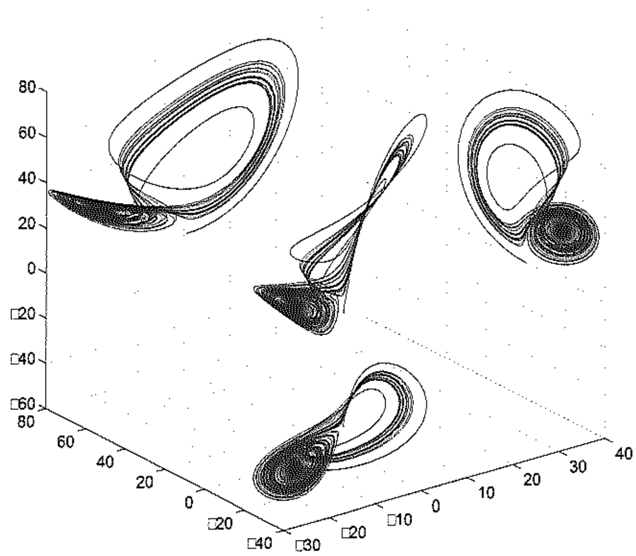

Fig. 5. Synchronization of the network in a chaotic attractor.

4 Conclusions

In this communication we illustrated the synchronization of a complex small-world network. The main contribution is that using a derivative coupling, a network with non identical systems in nodes can reach the synchronous behavior. We show that the network is leaded to an equilibrium point if the coupling factor is increased, but using the derivative coupling the synchronization in a chaotic attractor is obtained. The next step is apply this derivative coupling to scale-free networks which is still under study.

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